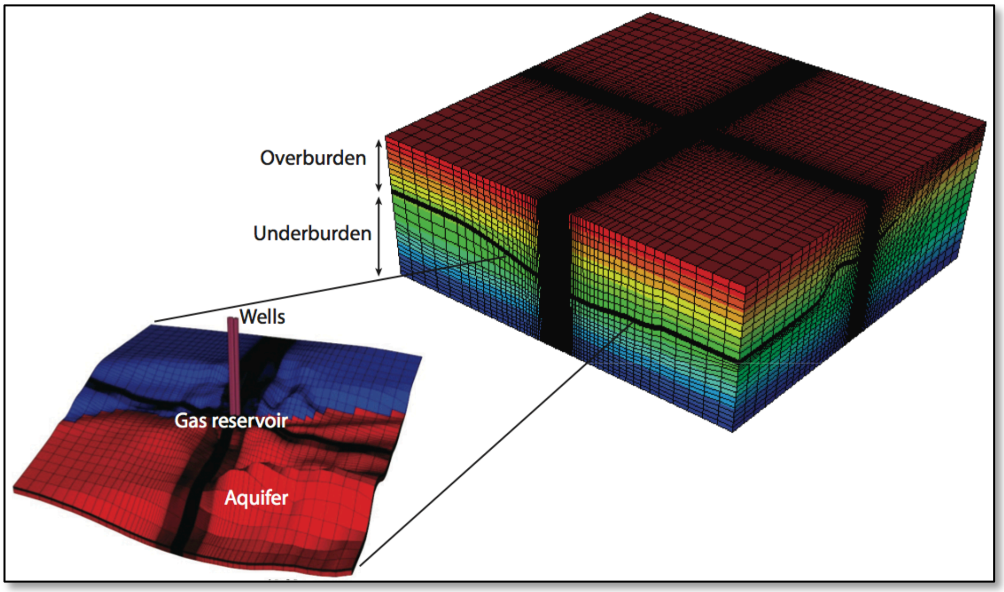
PTE 592 Computational Geomechanics



(Source: Jha et al., Int. J Numer. Anal. Methods Geomech., 2015)

Spring: Tue: 2:00 PM – 4:40 PM (Lecture) **Fri: 2:00 PM – 3:00 PM (Discussion)**
Location: TBA

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Course Description

Development of subsurface resources such as groundwater, hydrocarbons and geothermal energy requires an understanding of the fundamental natural processes of fluid flow and rock deformation in subsurface reservoirs. The two processes are governed by a coupled system of governing and constitutive equations which must be solved numerically on a computer for solution to real-world problems such as fluid production-induced and injection-induced subsidence and reservoir damage. This course covers physical, mathematical and simulation aspects of coupled fluid flow and geomechanics in subsurface reservoirs. Principles of continuum mechanics will be used to develop concepts of deformation, stress and poroelasticity. Conservation equations of mass and momentum applicable to fluid-saturated porous media will be derived. Finite element and finite volume methods for solution of those equations will be developed. Computer implementation of the numerical methods in Matlab (preferred), Python, C++, or Fortran will be emphasized. Students will also learn how to use ABAQUS for geomechanical simulations.

This course prepares professional-degree students for numerical modeling and simulation jobs in petroleum, environmental, and civil engineering sectors. The course is also suitable for PhD students interested in numerical simulations of partial differential equations and the subject of geomechanics.

Learning Objectives

At the end of this course, students will be able to

- define and calculate deformation, strain and stress in a body subjected to external forces
- derive conservation equations of fluid mass and solid momentum balance for a subsurface reservoir
- identify physical mechanisms and their mathematical forms that lead to coupling between fluid and solid behavior. Identify real-life implications of these coupling mechanisms.
- discretize the partial differential equations of mass-momentum balance in space using the finite element method
- integrate the equations in time using the Forward and Backward Euler methods
- program the finite element method in a computer for simultaneous solution of the two coupled problems of flow and mechanics
- list four different types of coupling schemes and their pros and cons for sequential solution of the flow-mechanics coupled problem
- discretize the partial differential equation of fluid mass balance using the finite volume method and the Backward Euler time integration
- program the sequential solution scheme in a computer with finite element for mechanics and finite volume for flow
- learn how to perform, and analyze the results of, a geomechanical simulation in ABAQUS

Prerequisite(s): None

Co-Requisite(s): None

Concurrent Enrollment: None

Recommended Preparation: Any combination of the following courses provide familiarity with some of the concepts covered in this course: MATH 225, MATH 245, PTE 500, PTE 508, AME 535a, AME 404, AME 507, CE 529

Course Notes

There is no prescribed textbook. Course notes and other materials will be handed out. When offered using DEN, the course material including lecture notes, homeworks and grades will be posted on the DEN webpage of the course. When offered without DEN support, the course material will be posted on Blackboard.

Reference Readings and Supplementary Materials

1. L. E. Malvern, Introduction to the Mechanics of a Continuous Medium, Prentice-Hall, Englewood Cliffs, NJ, 1969
2. J. Bear, Dynamics of Fluids in Porous Media, Dover Publications, NY, 1972
3. H. F. Wang, Theory of Linear Poroelasticity with Applications to Geomechanics and Hydrogeology, Princeton University Press, NJ, 2000
4. M. G. Larson and F. Bengzon, The Finite Element Method: Theory, Implementation, and Applications, Springer, 2013
5. H. F. Wang and M. P. Anderson, Introduction to Groundwater Modeling: Finite Difference and Finite Element Methods, Academic Press, 1995
6. M. D. Zoback, Reservoir Geomechanics, Cambridge University Press, 2007

Description and Evaluation of Assignments

Weekly homework assignments will be posted online after the lecture. Discussion sessions will demonstrate how to apply that week's concepts to solve numerical problems using coding. The homeworks will test theoretical understanding and computer implementation of the concepts and methods covered in the class. It will require numerical calculations using linear algebra, mathematical derivations using calculus, and computer programming. Some problems require schematic drawing to test the understanding of stress, strain, and deformation as well as plotting of numerical simulation results. Computer programs will be evaluated for accuracy and speed.

An in-class midterm examination will be conducted in Week 9 (see below) to test students' understanding of concepts covered in the course from Week 1 through 8. The final exam will be a week-long take-home coding project that students will submit through the DEN webpage or Blackboard depending on whether the DEN version is offered or not (See the final project description below). The final project will be due during the University-scheduled final exam period.

Class participation/in-class problem solving: During discussion sessions, students will be given 1-2 problems to solve in the class and submit their answers before they leave the class. Correct approach to solve the problem will be discussed near the end of the session. Student answers will be graded within two weeks as regular homeworks and comprise 10% of the grade.

Grading Breakdown

Assignment	**Points**	**% of Grade**
Class participation (in-class problem solving)	10	10
Homeworks (one-a-week)	40	40
Midterm (one in-class)	20	20
Final (take-home coding project)	30	30
TOTAL	100	100

Grading Scale

Course final grades will be determined using the following scale:

A 95-100
A- 90-94
B+ 84-89
B 78-83
B- 72-77
C+ 66-71
C 60-65
C- 54-59
D+ 48-53
D 42-47
D- 36-41
F 35 and below

Assignment Submission Policy

Homeworks are due at the beginning of the lecture class. Homeworks can be submitted electronically or in paper form. Computer programs must be submitted electronically and must run without any error to be counted towards grade.

Grading Timeline

Homeworks will be graded with feedback in less than two weeks from the date of submission. Their solutions will be discussed in class on the day of submission.

Additional Policies

Late submissions will cost a student 5% of the total grade of the submission for every late day.
It is expected that students attend every class and obtain prior approval for missing a class. All other changes in submission schedule require prior approval of the instructor.

Course Schedule: A Weekly Breakdown

	Topics/Daily Activities	**Discussion**	**Readings and Homework**	**Deliverable/Due Dates**
Week 1	Course introduction: Importance of geomechanics with application to oil and gas reservoir processes: compaction and subsidence, hydraulic fracturing, induced earthquakes, well failure. Role of computational methods in solving geomechanical problems	Refresher on vector algebra and vector calculus	Reference 3, chapter 1	
Week 2	Introduction to continuum mechanics. Kinematics of deformation, reference and spatial configurations. Deformation and strain, the Cauchy stress tensor and traction vector, principal stresses, stress invariants	Discussion of this week's lecture. Solve problems in class and learn how to calculate displacement, deformation, deformation gradient, Jacobian, the strain tensor.	Reference 1, chapter 3 and 4.	Homework 1/one week later
Week 3	Elastostatics. Derivation of momentum conservation laws, constitutive equation for linear elasticity, displacement formulation of the equilibrium equation, compatibility relations. Mohr's circle	In-class problem solving on calculation of principal stresses and Mohr's circle	Reference 1, chapter 5	Homework 2/one week later
Week 4	Analytical solution of the equilibrium equation using the Airy stress function. Formulation of boundary value problems	Practice the use of the Airy stress function in solving equilibrium problems analytically	Reference 4, chapter 3	Homework 3/one week later

	of 1D elasticity, the weak form.			
Week 5	Finite element method. Test functions, shape functions and their derivatives. Element integrations of weak form in element coordinates using Gauss quadrature. Global-to-element coordinate transformation. Element-stiffness matrix and load vector	Discussion of the Finite Element Method recipe. Draw shape functions and their derivatives. Practice Gauss quadrature method	Reference 3, chapter 7; Reference 4, chapter 4	Homework 4/two weeks later
Week 6	Formulation of the global matrix-vector problem and its computer implementation for 1D elasticity. Applying essential and natural boundary conditions.	Learn how to code the solution of 1D finite element problem in MATLAB	Reference 4, chapter 2	
Week 7	2D elasticity. Derivation of the equilibrium, constitutive, and compatibility equations for plane stress and plane strain.	Practice writing code to implement the 2D finite element method in MATLAB	Reference 3, chapter 3 and 4; Reference 5, chapter 4; Reference 2, chapter 6	Homework 5/one week later
Week 8	Fluid flow in deformable porous Media. Conservation laws, the effective stress, Darcy's law, drained and undrained behavior, poroelastic constants. Biot's theory of linear poroelasticity, analytical solution of Terzaghi's uniaxial compaction problem	Learn ABAQUS to setup a tutorial problem, solve it, and visualize the solutions	Reference 3, chapter 6.	Homework 6/one week later
Week 9	**Midterm**		Week 1 through 8 contents included in the exam.	
Week 10	Finite element approximation of the poroelasticity problem for simultaneous solution of nodal displacements and pressures in 1D	Practice on ABAQUS. Solve a 2D poroelastic problem e.g. consolidation	Reference 6, chapter 2 and 4	Homework 7/two weeks later

Week 11	Implicit and explicit time integration, accuracy and stability of forward and backward Euler integration schemes	Set up and solve consolidation problem in ABAQUS using 1st order, 2nd order and 3rd time integration methods	Reference 6, chapter 2 and 4	
Week 12	Finite volume approximation to solve the flow problem for cell-centered pressures. Derive the matrix-vector formulation in terms of transmissibility and compressibility matrices. Assembly of the global matrix-vector system. Apply fixed pressure and fixed flux boundary conditions. Solve at every time step for pressure evolution. Computer implementation	Practice writing finite volume code in MATLAB to solve a 2D porous media flow problem	Lecture notes	Homework 8/two weeks later
Week 13	Different coupling schemes for sequential solution of the flow and mechanics problems. Fixed-strain and fixed-stress formulation.	Practice joining the two codes—finite element for mechanics and finite volume for flow—to solve for displacements and pressure in a 2D domain	-- A coupled reservoir and geomechanical simulation system, Settari and Maurits, Soc. Pet. Eng. J 1998. -- Stability, Accuracy, and Efficiency of Sequential Methods for Coupled Flow and Geomechanics, Kim et al, Soc. Pet. Eng. J 2011	
Week 14	Computer implementation of the fixed-strain scheme.	Practice coding of fixed strain coupling scheme	Lecture notes	Homework 9/one week later
Week 15	Reservoir geomechanics. Tectonic stress regime, principal stresses, stresses induced by fluid production and injection. Rock failure. Tensile and shear failure modes	Calculate stress changes around and on a fault from production and injection of fluids.	Reference 7, chapter 1 and 2	No homework

FINAL	Solve a 2D poroelastic problem e.g. the Mandel problem			Date: For the date and time of the final for this class, consult the USC *Schedule of Classes* at www.usc.edu/soc.

Statement on Academic Conduct and Support Systems

Academic Conduct:

Plagiarism – presenting someone else's ideas as your own, either verbatim or recast in your own words – is a serious academic offense with serious consequences. Please familiarize yourself with the discussion of plagiarism in SCampus in Part B, Section 11, "Behavior Violating University Standards" policy.usc.edu/scampus-part-b. Other forms of academic dishonesty are equally unacceptable. See additional information in SCampus and university policies on scientific misconduct, policy.usc.edu/scientific-misconduct.

Support Systems:

Student Health Counseling Services - (213) 740-7711 – 24/7 on call
engemannshc.usc.edu/counseling
Free and confidential mental health treatment for students, including short-term psychotherapy, group counseling, stress fitness workshops, and crisis intervention.

National Suicide Prevention Lifeline - 1 (800) 273-8255 – 24/7 on call
suicidepreventionlifeline.org
Free and confidential emotional support to people in suicidal crisis or emotional distress 24 hours a day, 7 days a week.

Relationship and Sexual Violence Prevention Services (RSVP) - (213) 740-4900 – 24/7 on call
engemannshc.usc.edu/rsvp
Free and confidential therapy services, workshops, and training for situations related to gender-based harm.

Office of Equity and Diversity (OED) | Title IX - (213) 740-5086
equity.usc.edu, titleix.usc.edu
Information about how to get help or help a survivor of harassment or discrimination, rights of protected classes, reporting options, and additional resources for students, faculty, staff, visitors, and applicants. The university prohibits discrimination or harassment based on the following protected characteristics: race, color, national origin, ancestry, religion, sex, gender, gender identity, gender expression, sexual orientation, age, physical disability, medical condition, mental disability, marital status, pregnancy, veteran status, genetic information, and any other characteristic which may be specified in applicable laws and governmental regulations.

Bias Assessment Response and Support - (213) 740-2421
studentaffairs.usc.edu/bias-assessment-response-support
Avenue to report incidents of bias, hate crimes, and microaggressions for appropriate investigation and response.

The Office of Disability Services and Programs - (213) 740-0776

dsp.usc.edu
Support and accommodations for students with disabilities. Services include assistance in providing readers/notetakers/interpreters, special accommodations for test taking needs, assistance with architectural barriers, assistive technology, and support for individual needs.

USC Support and Advocacy - (213) 821-4710
studentaffairs.usc.edu/ssa
Assists students and families in resolving complex personal, financial, and academic issues adversely affecting their success as a student.
Diversity at USC - (213) 740-2101
diversity.usc.edu
Information on events, programs and training, the Provost's Diversity and Inclusion Council, Diversity Liaisons for each academic school, chronology, participation, and various resources for students.

USC Emergency - UPC: (213) 740-4321, HSC: (323) 442-1000 – 24/7 on call
dps.usc.edu, emergency.usc.edu
Emergency assistance and avenue to report a crime. Latest updates regarding safety, including ways in which instruction will be continued if an officially declared emergency makes travel to campus infeasible.

USC Department of Public Safety - UPC: (213) 740-6000, HSC: (323) 442-120 – 24/7 on call
dps.usc.edu
Non-emergency assistance or information.